CHAPTER – 2
RATIO AND PROPORTION

NOTES

➢ RATIO

- The ratio of two quantities of the same kind and in the same units is a comparision by division of the measure of two quantities. In other words, the ratio of two quantities of the same kind is the relation between their measures and determines how many times the one quantity is of the other quantity.

Notes:

1. The ratio of a to b is the fraction $\frac{a}{b}$ and is generally written as $a:b$. For the ratio $a:b$, the quantities a and b are called terms of the ratio. The former 'a' is called the first term or antecedent and the later 'b' is known as second term or consequent. The value of a ratio does not depend upon the nature of the quantities involved. It is an abstract number and has no unit.
2. The value of the ratio remains same, if both the antecedent and the consequent are multiplied or divided by the same non-zero quantity, because for any non- zero m, $a:b$ is the same as $ma:mb$ and $\frac{a}{b} = \frac{a/m}{b/m} \Rightarrow a:b$ is the same as $a/m : b/m$.

- **Compounded Ratio:** If two or more ratios are multiplied termwise, the ratio thus obtained is called their compounded ratio.

  Eg. The compounded ratio of $a:b$ and $c:d$ is $ac:bd$.

- **Duplicate Ratio:** It is the compounded ratio of two equal ratios. Thus, the duplicate ratio of $a:b$ is $aa:bb$ i.e. $a^2:b^2$.
- **Triplicate Ratio:** It is the compounded ratio of three equal ratios. Thus, the triplicate ratio of $a:b$ is $aaa:bbb$ i.e. $a^3:b^3$.
- **Sub-duplicate Ratio:** A ratio $x:y$ is sub-duplicate ratio of a ratio $a:b$ if the duplicate ratio of $x:y$ is $a:b$ i.e. $\frac{x^2}{y^2} = \frac{a}{b} \Rightarrow \frac{x}{y} = \frac{\sqrt{a}}{\sqrt{b}}$.

  Thus, the sub-duplicate ratio of $a:b$ is $\sqrt{a}:\sqrt{b}$.

- **Sub-triplicate Ratio:** The sub-triplicate ratio of $a:b$ is $\sqrt[3]{a}:\sqrt[3]{b}$.
- **Inverse Ratio:** The inverse ratio of $a:b$ is $b:a$.

PROPORTION

- Four quantities are said to be in proportion if the ratio of the first to the second is equal to the ratio of the third to the fourth. Thus, a, b, c and d are said to be in proportion if $a:b=c:d$ and we write $a:b::c:d$.

Notes:

1. If four quantities a, b, c and d are in proportion then we also say that they are proportional. The terms 'a and d' are called the extremes and 'b and c' are called the means. The last term 'd' is called the fourth proportional to a, b, c.
2. Four quantities a, b, c, d are in proportion if and only if the product of extremes is equal to the product of means i.e. $ad=bc$.

- **Continued Proportion:** Three quantities are said to be in continued proportion if the ratio of the first and the second is equal to the ratio of the second and third, i. e. a, b, c are in continued proportion if $a:b=b:c$

Note: If a, b, c are in continued proportion then b is called the mean proportional between a and c and is given by $\Rightarrow b^2=a\,c$ $i.e.\ b=\sqrt{ac}$ and 'c' is called the third proportional to a and b.

- **Rules of proportion**

If a, b, c, d are four quantities, then

(i) $a:b=c:d \Rightarrow b:a=d:c$ **(Invertendo)**

(ii) $a:b=c:d \Rightarrow a:c=b:d$ **(Alternendo)**

(iii) $a:b=c:d \Rightarrow (a+b):b=(c+d):d$ **(Componendo)**

(iv) $a:b=c:d \Rightarrow (a-b):b=(c-d):d$ **(Dividendo)**

(v) $a:b=c:d \Rightarrow (a+b):(a-b)=(c+d):(c-d)$ **(Componendo-Dividendo)**

(vi) $a:b=c:d \Rightarrow a:(a-b)=c:(c-d)$ **(Convertendo)**

(vii) $a:b=c:d \Rightarrow \frac{a}{b}=\frac{c}{d}=\frac{a+c}{b+d}$ **(Addendo)**

Note:

$If\ \frac{a}{b}=\frac{c}{d}=\frac{e}{f}= \ldots\ldots\ldots$, then each of the ratio $=\frac{a+c+e+\ldots\ldots}{b+d+f+\ldots\ldots}$

Theorem: If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}$, then each of the ratio $=\left(\frac{pa^n+qc^n+re^n}{pb^n+qd^n+rf^n}\right)^{\frac{1}{n}}$, where p, q, r, n are any quantities whatsoever.

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