DEPARTMENT OF EDUCATION (S)
Government of Manipur

CHAPTER 10
TRIGONOMETRY

Trigonometry – Study of relationship between the sides and angles of a triangle

- ***The sides of a right triangle:***

  (i) Opposite (ii) Adjacent (iii) Hypotenuse

- ***Trigonometric ratios of an acute angle of a right triangle:***

  (i) sine $= \frac{\text{opposite}}{\text{hypotenuse}}$ (iv) cosecant $= \frac{\text{hypotenuse}}{\text{opposite}}$

  (ii) cosine $= \frac{\text{adjacent}}{\text{hypotenuse}}$ (v) secant $= \frac{\text{hypotenuse}}{\text{adjacent}}$

  (iii) tangent $= \frac{\text{opposite}}{\text{adjacent}}$ (vi) cotangent $= \frac{\text{adjacent}}{\text{opposite}}$

Note:

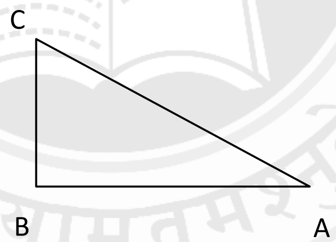
(i) Since the hypotenuse is the longest side of a right triangle, the value of sin A or cos A is always less than 1 for an acute angle A.

(ii) The values of trigonometric ratios of an angle do not vary with the size of the right triangle considered. In short, the trigonometric ratios of an angle are uniquely defined.

- ***Motivation of trigonometric ratios of 0^0 and 90^0:***

(i) In the right ΔABC right angled at B, let us suppose∠A= 0^0 (if possible).

Then,AB=AC and BC = 0,



Now, $\sin 0^0 = \frac{BC}{AC} = \frac{0}{AC} = 0$

$\cos 0^0 = \frac{AB}{AC} = 1$

$\tan 0^0 = \frac{BC}{AB} = \frac{0}{AB} = 0$

$\cot\ 0^0 = \frac{AB}{BC} = \frac{AB}{0}$ =undefined

$\sec 0^0 = \frac{AC}{AB} = 1$

$\text{cosec}\ 0^0 = \frac{AC}{BC} = \frac{AC}{0} =$ undefined

(ii)In the right ΔABC right angled at B, let us suppose∠A= 90^0 (if possible).

Then BC=AC and AB = 0,

Now, $\sin 90^0 = \frac{BC}{AC} = 1$

$\cos 90^0 = \frac{AB}{AC} = \frac{0}{AC} = 0$

$\tan 90^0 = \frac{BC}{AB} = \frac{BC}{0}$ =undefined

$\cot\ 90^0 = \frac{AB}{BC} = \frac{0}{BC} = 0$

$\sec 90^0 = \frac{AC}{AB} = \frac{AC}{0} =$ undefined

$\text{cosec}\ 90^0 = \frac{AC}{BC} = 1$

- ***Trigonometric ratios of*** 30^0 ***and*** 60^0***:***

Let us consider an equilateral ΔABC.

Then, ∠A=∠B=∠C= $\mathbf{60^0}$

and AB = BC = AC = $2k$ (say)

Let us draw AD perpendicular to BC meeting BC at D.

Then, BD = CD = k and ∠BAD = ∠CAD = 30^0 (by the property of an equilateral triangle).

Now, in the right ΔABD,

$$AD^2 + BD^2 = AB^2 \quad \text{(by Pythagoras theorem)}$$

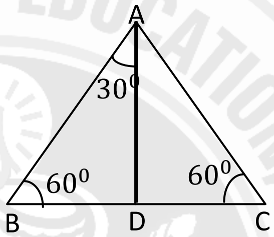
$$\Rightarrow AD^2 + k^2 = (2k)^2$$

$$\Rightarrow AD^2 = 4k^2 - k^2$$

$$\Rightarrow AD^2 = 3k^2$$

$$\Rightarrow AD = \sqrt{3k^2}$$

$$\Rightarrow AD = \sqrt{3}\,k$$



(i) In the right ΔABD, considering ∠BAD = 30^0, we have

$$\sin 30^0 = \frac{BD}{AB} = \frac{k}{2k} = \frac{1}{2}$$

$$\cos 30^0 = \frac{AD}{AB} = \frac{\sqrt{3}\,k}{2k} = \frac{\sqrt{3}}{2}$$

$$\tan 30^0 = \frac{BD}{AD} = \frac{k}{\sqrt{3}\,k} = \frac{1}{\sqrt{3}}$$

$$\cot 30^0 = \frac{AD}{BD} = \frac{\sqrt{3}\,k}{k} = \sqrt{3}$$

$$\sec 30^0 = \frac{AB}{AD} = \frac{2k}{\sqrt{3}\,k} = \frac{2}{\sqrt{3}}$$

$$\operatorname{cosec} 30^0 = \frac{AB}{BD} = \frac{2k}{k} = 2$$

(ii)In the right ΔABD, considering ∠ABD = 60^0, we have

$$\sin 60^0 = \frac{AD}{AB} = \frac{\sqrt{3}\,k}{2k} = \frac{\sqrt{3}}{2}$$

$$\cos 60^0 = \frac{BD}{AB} = \frac{k}{2k} = \frac{1}{2}$$

$$\tan 60^0 = \frac{AD}{BD} = \frac{\sqrt{3}\,k}{k} = \sqrt{3}$$

$$\cot 60^0 = \frac{BD}{AD} = \frac{k}{\sqrt{3}\,k} = \frac{1}{\sqrt{3}}$$

$$\sec 60^0 = \frac{AB}{BD} = \frac{2k}{k} = 2$$

$$\operatorname{cosec} 60^0 = \frac{AB}{AD} = \frac{2k}{\sqrt{3}\,k} = \frac{2}{\sqrt{3}}$$

- ***Trigonometric ratios of*** 45^0***:***

In the right ΔABC, right angled at B, if one angle is 45^0, then the other angle is also 45^0, i.e. if ∠A=∠C= 45^0.

So, AB = BC [∵∠A=∠C]

Suppose, AB = BC = k

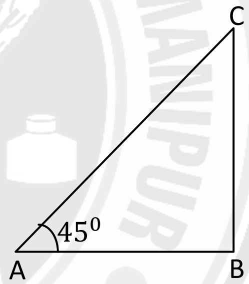
Now, $AC^2 = AB^2 + BC^2$ (by Pythagoras theorem)

$$= k^2 + k^2$$

$$= 2k^2$$

$$\therefore AC = \sqrt{2}k$$

Then, considering ∠A= 45^0, using definitions of trigonometric ratios, we have,



$$\sin 45^0 = \frac{BC}{AC} = \frac{k}{\sqrt{2}k} = \frac{1}{\sqrt{2}}$$

$$\cos 45^0 = \frac{AB}{AC} = \frac{k}{\sqrt{2}k} = \frac{1}{\sqrt{2}}$$

$$\tan 45^0 = \frac{BC}{AB} = \frac{k}{k} = 1$$

$$\cot 45^0 = \frac{AB}{BC} = \frac{k}{k} = 1$$

$$\sec 45^0 = \frac{AC}{AB} = \frac{\sqrt{2}k}{k} = \sqrt{2}$$

$$\text{cosec } 45^0 = \frac{AC}{BC} = \frac{\sqrt{2}k}{k} = \sqrt{2}$$

- ***Trigonometric Tables:***

∠A	0^0	30^0	45^0	60^0	90^0
sin A	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos A	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan A	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	undefined
cot A	undefined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
sec A	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	undefined
cosec A	undefined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

- ***Relationship between the Trigonometric Ratios:***
  - ***Reciprocal Relations***

(i) $\sin A = \frac{1}{cosec\ A}$ $cosec\ A = \frac{1}{\sin A}$

(ii) $\cos A = \frac{1}{\sec A}$ $\sec A = \frac{1}{\cos A}$

(iii) $\tan A = \frac{1}{\cot A}$ $\cot A = \frac{1}{\tan A}$

  - ***Quotient Relations***

(i) $\tan A = \frac{\sin A}{\cos A}$ (ii) $\cot A = \frac{\cos A}{\sin A}$

  - ***Pythagorean Relations***

(i) $\left.\begin{array}{l}\sin^2 A + \cos^2 A = 1\\ \sin^2 A = 1 - \cos^2 A\\ \cos^2 A = 1 - \sin^2 A\end{array}\right\} \quad 0 \le A \le 90^0$

(ii) $\left.\begin{array}{l}1 + \tan^2 A = \sec^2 A\\ \sec^2 A - \tan^2 A = 1\\ \tan^2 A = \sec^2 A - 1\end{array}\right\} \quad 0 \le A < 90^0$

(iii) $\left.\begin{array}{l}1 + cot^2 A = cosec^2 A\\ cosec^2 A - cot^2 A = 1\\ cot^2 A = cosec^2 A - 1\end{array}\right\} 0 < A \le 90^0$

- Establish the relations:

(i) $\sin^2 A + \cos^2 A = 1$

(ii) $1 + \tan^2 A = \sec^2 A$

(iii) $1 + cot^2 A = cosec^2 A$

Ans: (i) In a right triangle, we know,

$\text{Opposite}^2 + \text{Adjacent}^2 = \text{Hypotenuse}^2$ (by Pythagoras Theorem)

$\Rightarrow \frac{opposite^2}{hypotenuse^2} + \frac{adjacent^2}{hypotenuse^2} = \frac{hypotenuse^2}{hypotenuse^2}$

$\Rightarrow sin^2 A + cos^2 A = 1$

(ii) In a right triangle, we know,

$\text{Opposite}^2 + \text{Adjacent}^2 = \text{Hypotenuse}^2$ (by Pythagoras Theorem)

$\Rightarrow \frac{opposite^2}{adjacent^2} + \frac{adjacent^2}{adjacent^2} = \frac{hypotenuse^2}{adjacent^2}$

$\Rightarrow tan^2 A + 1 = sec^2 A$

$\Rightarrow 1 + tan^2 A = sec^2 A$

(iii) In a right triangle, we know,

$\text{Opposite}^2 + \text{Adjacent}^2 = \text{Hypotenuse}^2$ (by Pythagoras Theorem)

$\Rightarrow \frac{opposite^2}{opposite^2} + \frac{adjacent^2}{opposite^2} = \frac{hypotenuse^2}{opposite^2}$

$\Rightarrow 1 + cot^2 A = cosec^2 A$

- ***Trigonometric Identity***

An equation involving trigonometric ratios of an angle is called a trigonometric identity if it is true for all admissible values of the angle involved.

Example:- $cos^2 A - cos\ A(cos\ A - 1)$

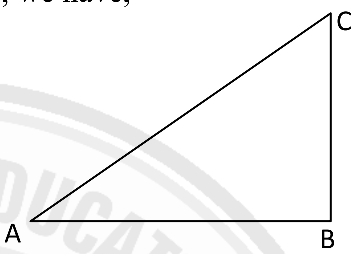
➢ Trigonometric Ratios of Complementary Angles

In the right ΔABC, right angled at B, ∠A and ∠C are the acute angles.

Hence ∠A+∠C = 90^0 i.e. they are complementary.

Then ∠C = $90^0 - \angle A$.

Now, by using definitions of trigonometric ratios, we have,



(i) $\sin A = \frac{BC}{AC} = \cos C = \cos(90^0 - A)$

(ii) $\cos A = \frac{AB}{AC} = \sin C = \sin(90^0 - A)$

(iii) $\tan A = \frac{BC}{AB} = \cot C = \cot(90^0 - A)$

(iv) $\cot A = \frac{AB}{BC} = \tan C = \tan(90^0 - A)$

(v) $\sec A = \frac{AC}{AB} = \text{cosec } C = \text{cosec}(90^0 - A)$

(vi) $\text{cosec } A = \frac{AC}{BC} = \sec C = \sec(90^0 - A)$

Height & Distance

- **Angle of elevation:** The angle of elevation of a point observed is the angle formed by the line of sight with horizontal, when the point being observed is above the horizontal through the eye.
- **Angle of depression:** The angle of depression of a point observed is the angle formed by the line with the horizontal when the point observed is below the horizontal through eye.

Note:

(i) If height of the object is more than the length of its shadow, the altitude of the sun is 60^0.

(ii) If the height of the object is equal to the length of its shadow, the altitude of the sun is 45^0.

(iii) If the height of the object is less than the length of its shadow, the altitude of the sun is 30^o.

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